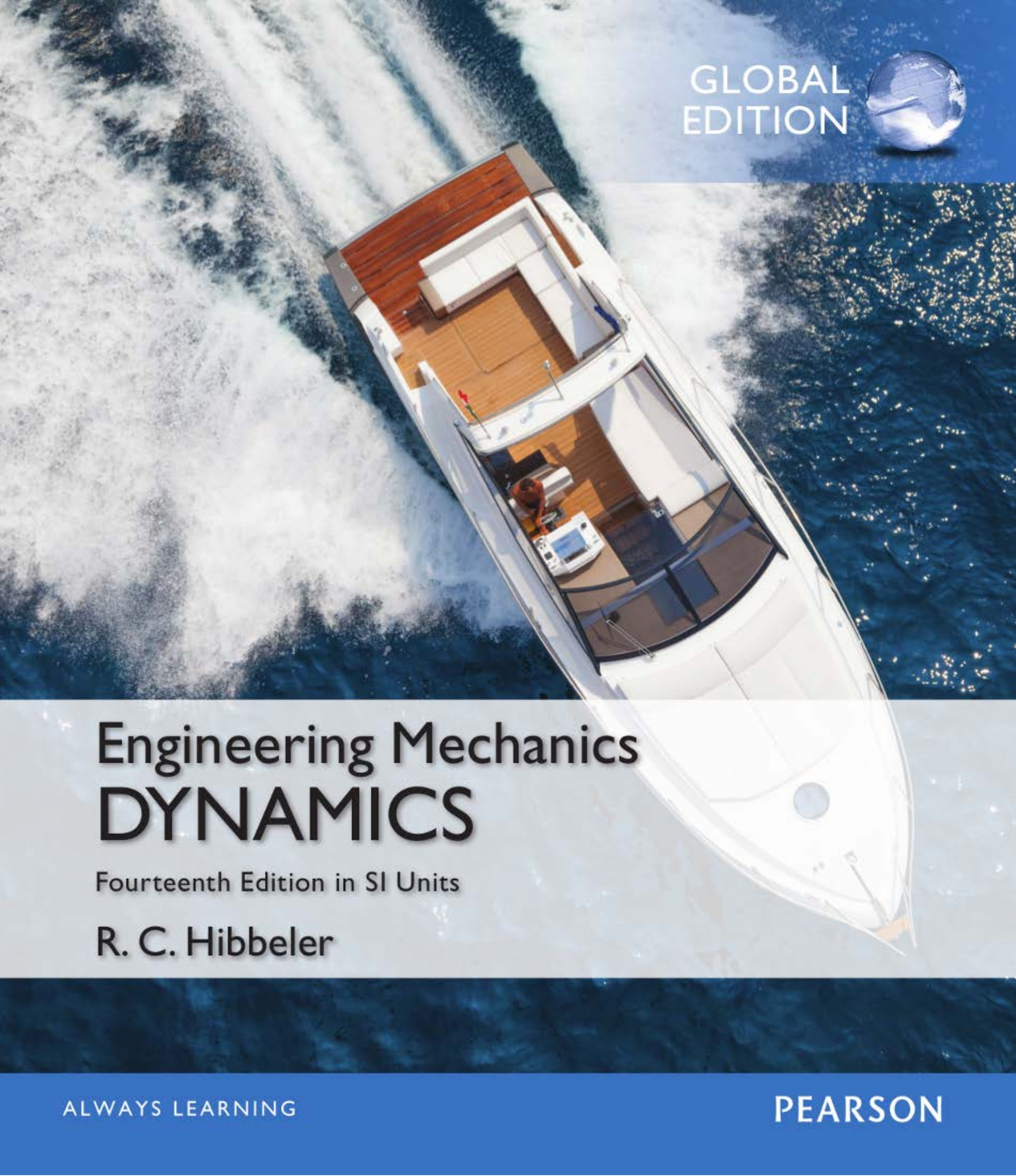

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Engineering Mechanics
DYNAMICS

Fourteenth Edition in SI Units

R. C. Hibbeler

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DYNAMICS

FOURTEENTH EDITION IN SI UNITS

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SI Conversion by
Kai Beng Yap

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To the Student

With the hope that this work will stimulate
an interest in Engineering Mechanics
and provide an acceptable guide to its understanding.

PREFACE

The main purpose of this book is to provide the student with a clear and thorough presentation of the theory and application of engineering mechanics. To achieve this objective, this work has been shaped by the comments and suggestions of hundreds of reviewers in the teaching profession, as well as many of the author's students.

New to this Edition

Preliminary Problems. This new feature can be found throughout the text, and is given just before the Fundamental Problems. The intent here is to test the student's conceptual understanding of the theory. Normally the solutions require little or no calculation, and as such, these problems provide a basic understanding of the concepts before they are applied numerically. All the solutions are given in the back of the text.

Expanded Important Points Sections. Summaries have been added which reinforces the reading material and highlights the important definitions and concepts of the sections.

Re-writing of Text Material. Further clarification of concepts has been included in this edition, and important definitions are now in boldface throughout the text to highlight their importance.

End-of-the-Chapter Review Problems. All the review problems now have solutions given in the back, so that students can check their work when studying for exams, and reviewing their skills when the chapter is finished.

New Photos. The relevance of knowing the subject matter is reflected by the real-world applications depicted in the over 30 new or updated photos placed throughout the book. These photos generally are used to explain how the relevant principles apply to real-world situations and how materials behave under load.

New Problems. There are approximately 30% new problems that have been added to this edition, which involve applications to many different fields of engineering.

Hallmark Features

Besides the new features mentioned above, other outstanding features that define the contents of the text include the following.

Organization and Approach. Each chapter is organized into well-defined sections that contain an explanation of specific topics, illustrative example problems, and a set of homework problems. The topics within each section are placed into subgroups defined by boldface titles. The purpose of this is to present a structured method for introducing each new definition or concept and to make the book convenient for later reference and review.

Chapter Contents. Each chapter begins with an illustration demonstrating a broad-range application of the material within the chapter. A bulleted list of the chapter contents is provided to give a general overview of the material that will be covered.

Emphasis on Free-Body Diagrams. Drawing a free-body diagram is particularly important when solving problems, and for this reason this step is strongly emphasized throughout the book. In particular, special sections and examples are devoted to show how to draw free-body diagrams. Specific homework problems have also been added to develop this practice.

Procedures for Analysis. A general procedure for analyzing any mechanical problem is presented at the end of the first chapter. Then this procedure is customized to relate to specific types of problems that are covered throughout the book. This unique feature provides the student with a logical and orderly method to follow when applying the theory. The example problems are solved using this outlined method in order to clarify its numerical application. Realize, however, that once the relevant principles have been mastered and enough confidence and judgment have been obtained, the student can then develop his or her own procedures for solving problems.

Important Points. This feature provides a review or summary of the most important concepts in a section and highlights the most significant points that should be realized when applying the theory to solve problems.

Fundamental Problems. These problem sets are selectively located just after most of the example problems. They provide students with simple applications of the concepts, and therefore, the chance to develop their problem-solving skills before attempting to solve any of the standard problems that follow. In addition, they can be used for preparing for exams.

Conceptual Understanding. Through the use of photographs placed throughout the book, theory is applied in a simplified way in order to illustrate some of its more important conceptual features and instill the physical meaning of many of the terms used in the equations. These simplified applications increase interest in the subject matter and better prepare the student to understand the examples and solve problems.

Homework Problems. Apart from the Fundamental and Conceptual type problems mentioned previously, other types of problems contained in the book include the following:

- **Free-Body Diagram Problems.** Some sections of the book contain introductory problems that only require drawing the free-body diagram for the specific problems within a problem set. These assignments will impress upon the student the importance of mastering this skill as a requirement for a complete solution of any equilibrium problem.

- **General Analysis and Design Problems.** The majority of problems in the book depict realistic situations encountered in engineering practice. Some of these problems come from actual products used in industry. It is hoped that this realism will both stimulate the student's interest in engineering mechanics and provide a means for developing the skill to reduce any such problem from its physical description to a model or symbolic representation to which the principles of mechanics may be applied.

  An attempt has been made to arrange the problems in order of increasing difficulty except for the end of chapter review problems, which are presented in random order.

- **Computer Problems.** An effort has been made to include some problems that may be solved using a numerical procedure executed on either a desktop computer or a programmable pocket calculator. The intent here is to broaden the student's capacity for using other forms of mathematical analysis without sacrificing the time needed to focus on the application of the principles of mechanics. Problems of this type, which either can or must be solved using numerical procedures, are identified by a "square" symbol (■) preceding the problem number.

The many homework problems in this edition, have been placed into two different categories. Problems that are simply indicated by a problem number have an answer and in some cases an additional numerical result given in the back of the book. An asterisk (*) before every fourth problem number indicates a problem without an answer.

Accuracy. As with the previous editions, apart from the author, the accuracy of the text and problem solutions has been thoroughly checked by four other parties: Scott Hendricks, Virginia Polytechnic Institute and State University; Karim Nohra, University of South Florida; Kurt Norlin, Bittner Development Group; and finally Kai Beng, a practicing engineer, who in addition to accuracy review provided suggestions for problem development.

Animations. On the Companion Website are seven animations identified as fundamental engineering mechanics concepts. The animations, flagged by a film icon, help students visualize the relation between mathematical explanation and real structure, breaking down complicated sequences and showing how free-body diagrams can be derived. These animations lend a graphic component in tutorials and lectures, assisting instructors in demonstrating the teaching of concepts with greater ease and clarity.

Each animation is flagged by a film icon.

254 CHAPTER 15 KINETICS OF A PARTICLE: IMPULSE AND MOMENTUM

15

Please refer to the Companion Website for the animation: *Impact of 2 Sliding Masses*

15.3 Conservation of Linear Momentum for a System of Particles

When the sum of the *external impulses* acting on a system of particles is *zero*, Eq. 15–6 reduces to a simplified form, namely,

$$\boxed{\Sigma m_i(\mathbf{v}_i)_1 = \Sigma m_i(\mathbf{v}_i)_2} \qquad (15\text{–}8)$$

This equation is referred to as the *conservation of linear momentum*. It states that the total linear momentum for a system of particles remains constant during the time period t_1 to t_2. Substituting $m\mathbf{v}_G = \Sigma m_i\mathbf{v}_i$ into Eq. 15–8, we can also write

$$(\mathbf{v}_G)_1 = (\mathbf{v}_G)_2 \qquad (15\text{–}9)$$

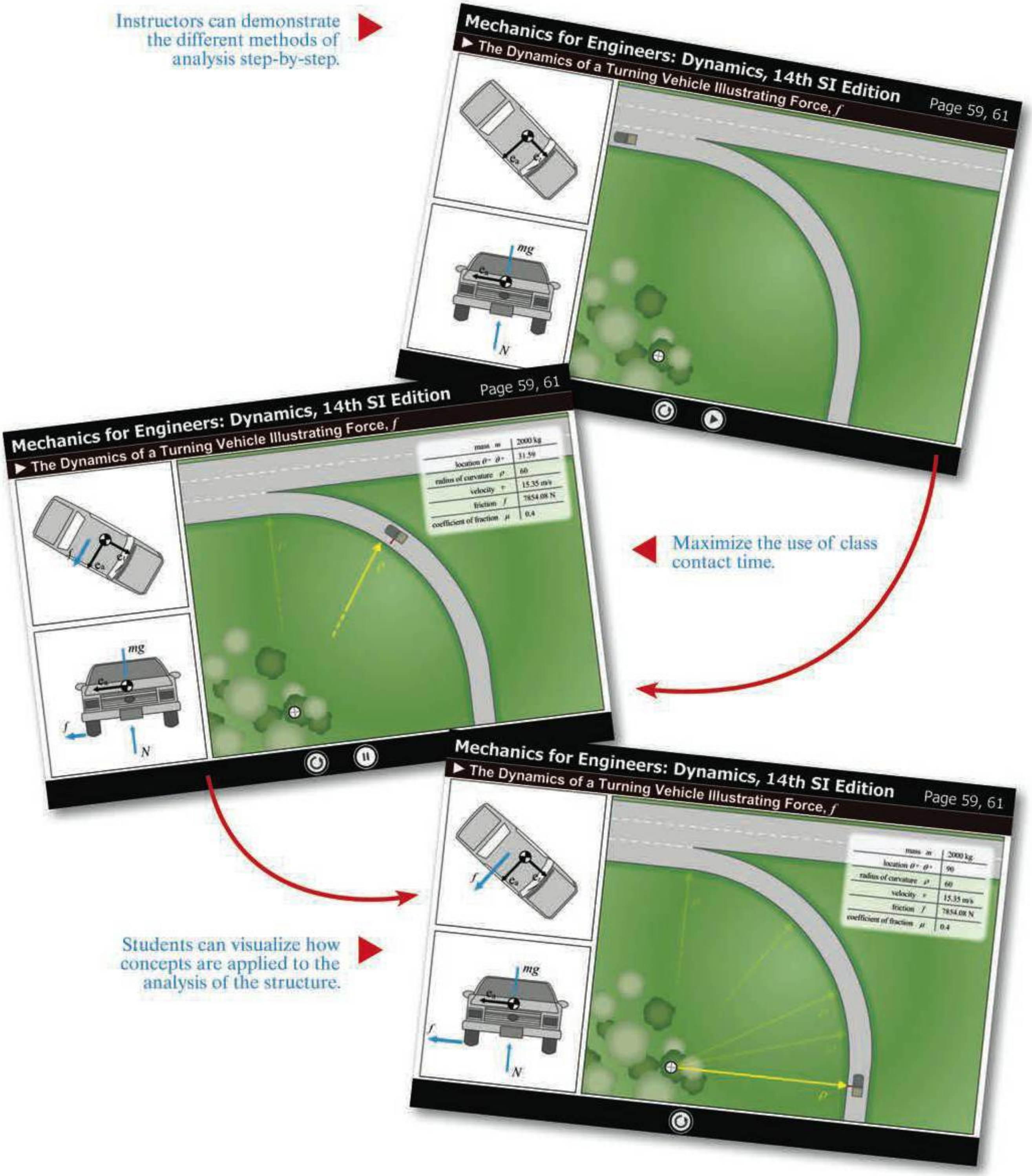
which indicates that the velocity $\mathbf{v}_G$ of the mass center for the system of particles does not change if no external impulses are applied to the system.

The conservation of linear momentum is often applied when particles collide or interact. For application, a careful study of the free-body diagram for the *entire* system of particles should be made in order to identify the forces which create either external or internal impulses and thereby determine in what direction(s) linear momentum is conserved. As stated earlier, the *internal impulses* for the system will always cancel out, since they occur in equal but opposite collinear pairs. If the time period over which the motion is studied is *very short*, some of the external impulses may also be neglected or considered approximately equal to zero. The forces causing these negligible impulses are called *nonimpulsive forces*. By comparison, forces which are very large and act for a very short period of time produce a significant change in momentum and are called *impulsive forces*. They, of course, cannot be neglected in the impulse–momentum analysis.

Impulsive forces normally occur due to an explosion or the striking of one body against another, whereas nonimpulsive forces may include the weight of a body, the force imparted by a slightly deformed spring having a relatively small stiffness, or for that matter, any force that is very small compared to other larger (impulsive) forces. When making this distinction between impulsive and nonimpulsive forces, it is important to realize that this only applies during the time t_1 to t_2. To illustrate, consider the effect of striking a tennis ball with a racket as shown in the photo. During the *very short* time of interaction, the force of the racket on the ball is impulsive since it changes the ball's momentum drastically. By comparison, the ball's weight will have a negligible effect on the change

The hammer in the top photo applies an impulsive force to the stake. During this extremely short time of contact the weight of the stake can be considered nonimpulsive, and provided the stake is driven into soft ground, the impulse of the ground acting on the stake can also be considered nonimpulsive. By contrast, if the stake is used in a concrete chipper to break concrete, then two impulsive forces act on the stake: one at its top due to the chipper and the other on its bottom due to the rigidity of the concrete.

Instructors can demonstrate the different methods of analysis step-by-step.



Mechanics for Engineers: Dynamics, 14th SI Edition

▶ The Dynamics of a Turning Vehicle Illustrating Force, f

Page 59, 61

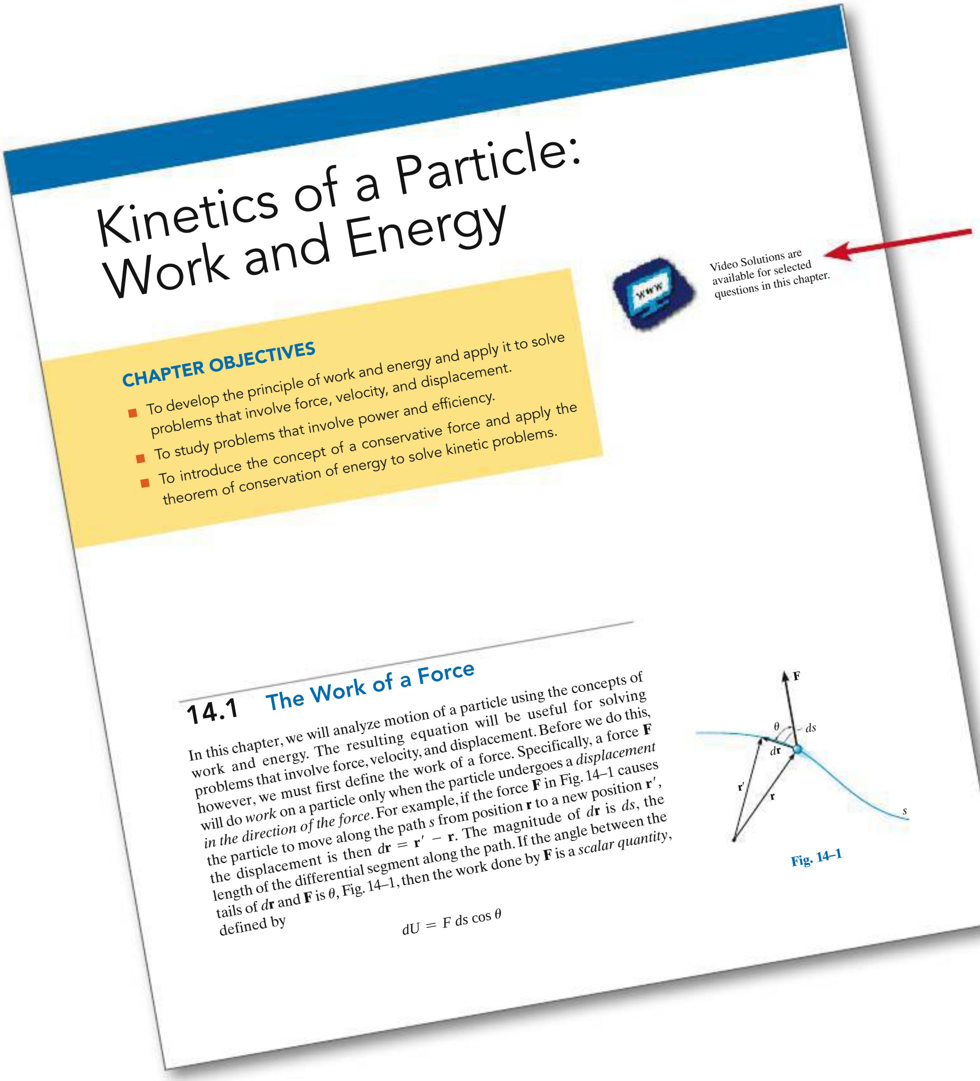
mass m	2000 kg
location $\theta = \theta_*$	31.59
radius of curvature ρ	60
velocity v	15.35 m/s
friction f	7854.08 N
coefficient of fraction μ	0.4

mass m	2000 kg
location $\theta = \theta_*$	90
radius of curvature ρ	60
velocity v	15.35 m/s
friction f	7854.08 N
coefficient of fraction μ	0.4

Maximize the use of class contact time.

Students can visualize how concepts are applied to the analysis of the structure.

Video Solutions. An invaluable resource in and out of the classroom, these complete solution walkthroughs of representative homework problems from each chapter offer fully worked solutions, self-paced instruction, and 24/7 accessibility. Lecturers and students can harness this resource to gain independent exposure to a wide range of examples applying formulas to actual structures.



Kinetics of a Particle: Work and Energy

Video Solutions are available for selected questions in this chapter.

CHAPTER OBJECTIVES

- To develop the principle of work and energy and apply it to solve problems that involve force, velocity, and displacement.
- To study problems that involve power and efficiency.
- To introduce the concept of a conservative force and apply the theorem of conservation of energy to solve kinetic problems.

14.1 The Work of a Force

In this chapter, we will analyze motion of a particle using the concepts of work and energy. The resulting equation will be useful for solving problems that involve force, velocity, and displacement. Before we do this, however, we must first define the work of a force. Specifically, a force **F** will do *work* on a particle only when the particle undergoes a *displacement in the direction of the force*. For example, if the force **F** in Fig. 14–1 causes the particle to move along the path s from position $\mathbf{r}$ to a new position $\mathbf{r}'$, the displacement is then $d\mathbf{r} = \mathbf{r}' - \mathbf{r}$. The magnitude of $d\mathbf{r}$ is ds, the length of the differential segment along the path. If the angle between the tails of $d\mathbf{r}$ and **F** is θ, Fig. 14–1, then the work done by **F** is a *scalar quantity*, defined by

$$dU = F\,ds\cos\theta$$

Fig. 14–1

Video solutions are available for certain questions.

Independent video replays of a lecturer's explanation reinforces students' understanding.

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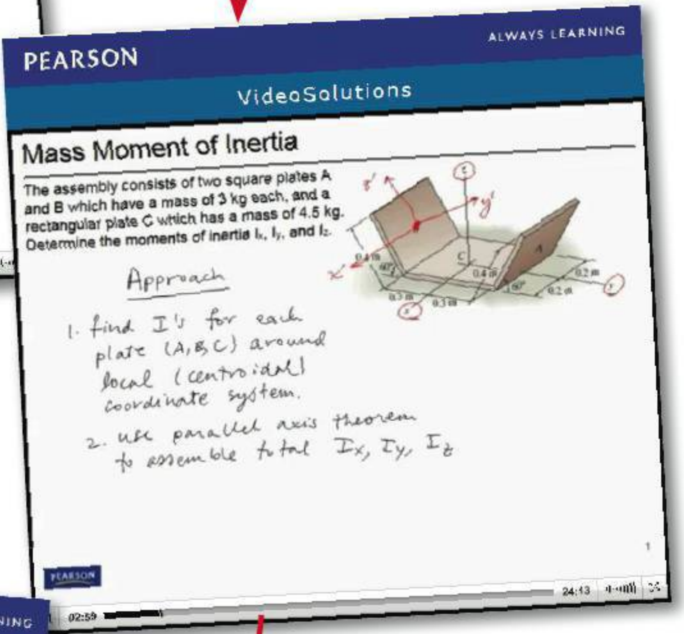
Video Solution

Mass Moment of Inertia

This problem, although it looks fairly innocent, requires us to use essentially all of our inertia transformation tools for its solution. First, we break the structure into composite parts, exactly as we have done for moment of inertia calculations starting in statics. Then, for each piece, we apply coordinate rotation equations for the two centroidal coordinate systems we define for each piece. Finally, we apply the parallel axis theorem to translate the moments of inertia onto the global (x,y,z) axes defined at the centroid of the entire structure. This problem provides excellent practice and requires great attention to detail for its solution.



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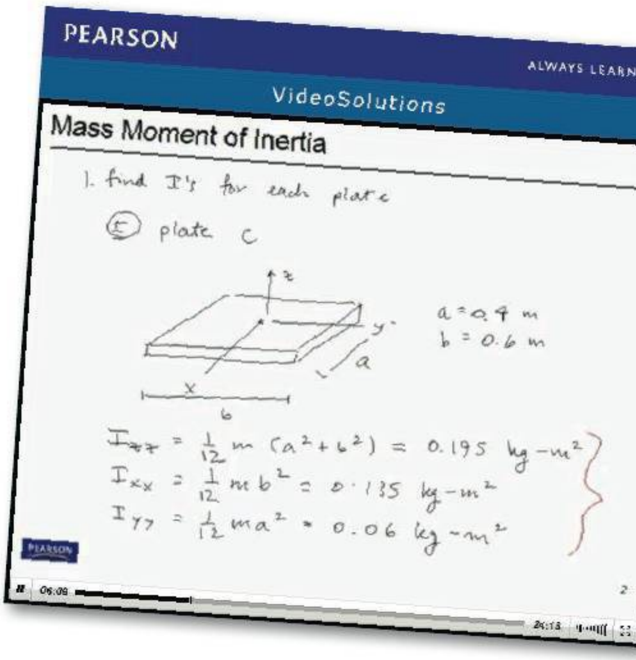
Mass Moment of Inertia

The assembly consists of two square plates A and B which have a mass of 3 kg each, and a rectangular plate C which has a mass of 4.5 kg. Determine the moments of inertia I_x, I_y, and I_z.

Approach

1. find I's for each plate (A,B,C) around local (centroidal) coordinate system.
2. use parallel axis theorem to assemble total I_x, I_y, I_z

Reduces lecturers' time spent in repetitive explanation of concepts and applications.



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Mass Moment of Inertia

1. find I's for each plate

ⓒ plate C

$a = 0.4$ m

$b = 0.6$ m

$I_{zz} = \frac{1}{12} m (a^2 + b^2) = 0.195$ kg-m^2

$I_{xx} = \frac{1}{12} m b^2 = 0.135$ kg-m^2

$I_{yy} = \frac{1}{12} m a^2 = 0.06$ kg-m^2

Flexible resource for students where they can learn at a comfortable pace without relying too much on their instuctors.

Contents

The book is divided into 11 chapters, in which the principles are first applied to simple, then to more complicated situations.

The kinematics of a particle is discussed in Chapter 12, followed by a discussion of particle kinetics in Chapter 13 (Equation of Motion), Chapter 14 (Work and Energy), and Chapter 15 (Impulse and Momentum). The concepts of particle dynamics contained in these four chapters are then summarized in a "review" section, and the student is given the chance to identify and solve a variety of problems. A similar sequence of presentation is given for the planar motion of a rigid body: Chapter 16 (Planar Kinematics), Chapter 17 (Equations of Motion), Chapter 18 (Work and Energy), and Chapter 19 (Impulse and Momentum), followed by a summary and review set of problems for these chapters.

If time permits, some of the material involving three-dimensional rigid-body motion may be included in the course. The kinematics and kinetics of this motion are discussed in Chapters 20 and 21, respectively. Chapter 22 (Vibrations) may be included if the student has the necessary mathematical background. Sections of the book that are considered to be beyond the scope of the basic dynamics course are indicated by a star (★) and may be omitted. Note that this material also provides a suitable reference for basic principles when it is discussed in more advanced courses. Finally, Appendix A provides a list of mathematical formulas needed to solve the problems in the book, Appendix B provides a brief review of vector analysis, and Appendix C reviews application of the chain rule.

Alternative Coverage. At the discretion of the instructor, it is possible to cover Chapters 12 through 19 in the following order with no loss in continuity: Chapters 12 and 16 (Kinematics), Chapters 13 and 17 (Equations of Motion), Chapter 14 and 18 (Work and Energy), and Chapters 15 and 19 (Impulse and Momentum).

Acknowledgments

The author has endeavored to write this book so that it will appeal to both the student and instructor. Through the years, many people have helped in its development, and I will always be grateful for their valued suggestions and comments. Specifically, I wish to thank all the individuals who have contributed their comments relative to preparing the fourteenth edition of this work, and in particular, R. Bankhead of Highline Community College, K. Cook-Chennault of Rutgers, the State University of New Jersey, E. Erisman, College of Lake County Illinois, M. Freeman of the University of Alabama, H. Lu of University of Texas at Dallas, J. Morgan of Texas A & M University, R. Neptune of the University of Texas, I. Orabi of the University of New Haven, T. Tan, University of Memphis, R. Viesca of Tufts University, and G. Young, Oklahoma State University.

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I would greatly appreciate hearing from you if at any time you have any comments, suggestions, or problems related to any matters regarding this edition.

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Global Edition

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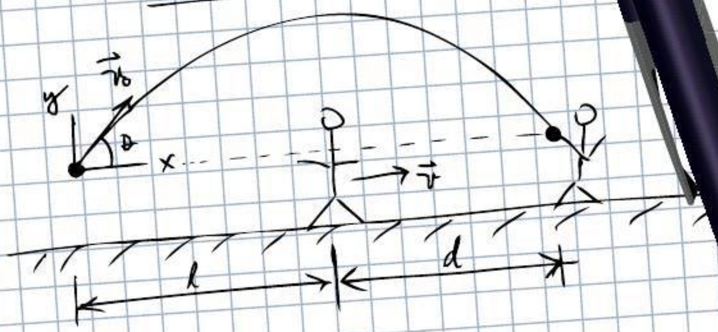
your **work**...

PART A



Given : $v = 7.000\,m/s$; $t = 2.000\,s$; $l = 18.00\,m$

$d = v \cdot t \Rightarrow d = (7.000\,m/s)(2.000\,s) = 14.00\,m$

$x = l + d \Rightarrow x = 18.00\,m + 14.00\,m = 32.00\,m$

$g = 9.807\,m/s^2$

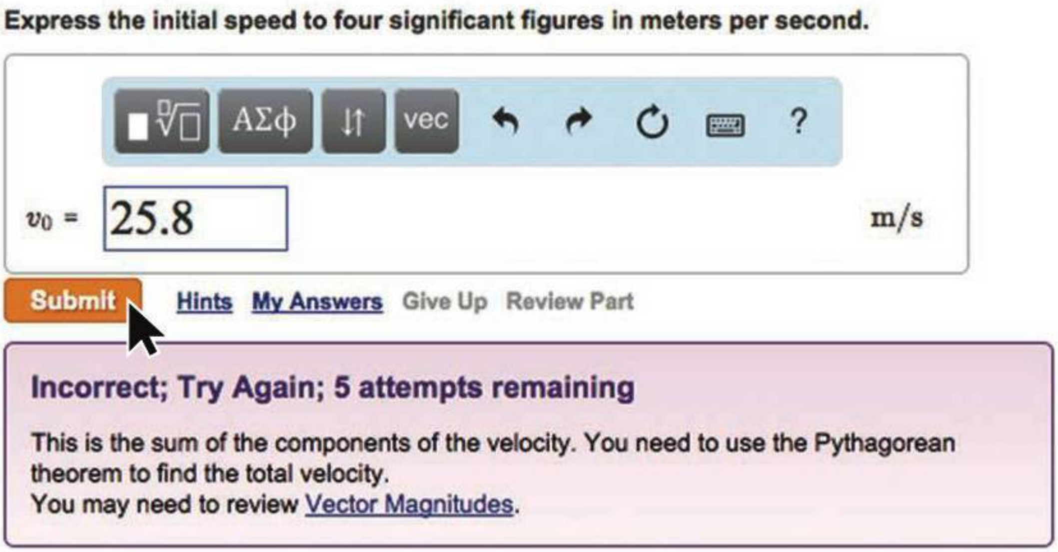
$v_{0x} = \frac{x}{t} = \frac{32.00\,m}{2.000\,s} = 16.00\,m/s$ (COMP. X)

$v_{0y} = \frac{1}{2} g t = \frac{1}{2}(9.807\,m/s^2)(2.000\,s) = 9.80\,m/s$ (COMP. Y)

$v_0 = v_{0x} + v_{0y} = 16.00\,m/s + 9.80\,m/s = 25.80\,m/s$ (TOTAL)

$\boxed{v_0 = 25.80\,m/s}$

your answer **specific feedback**



Express the initial speed to four significant figures in meters per second.

v_0 = 25.8 m/s

Incorrect; Try Again; 5 attempts remaining

This is the sum of the components of the velocity. You need to use the Pythagorean theorem to find the total velocity.
You may need to review Vector Magnitudes.

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Resources for Instructors

- **MasteringEngineering.** This online Tutorial Homework program allows you to integrate dynamic homework with automatic grading and adaptive tutoring. MasteringEngineering allows you to easily track the performance of your entire class on an assignment-by-assignment basis, or the detailed work of an individual student.

- **Instructor's Solutions Manual.** This supplement provides complete solutions supported by problem statements and problem figures. The fourteenth edition manual was revised to improve readability and was triple accuracy checked. The Instructor's Solutions Manual is available on the Pearson Higher Education website: www.pearsonglobaleditions.com.

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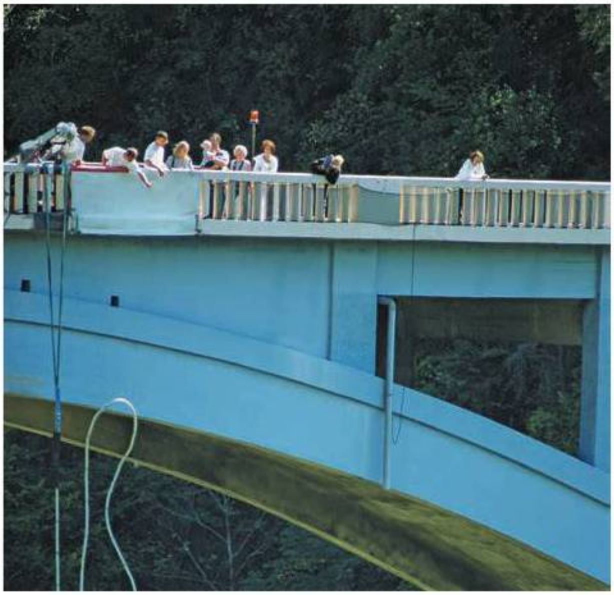

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ENGINEERING MECHANICS

DYNAMICS

FOURTEENTH EDITION IN SI UNITS

Chapter 12

(© Lars Johansson/Fotolia)

Although each of these boats is rather large, from a distance their motion can be analyzed as if each were a particle.

Kinematics of a Particle

CHAPTER OBJECTIVES

- To introduce the concepts of position, displacement, velocity, and acceleration.
- To study particle motion along a straight line and represent this motion graphically.
- To investigate particle motion along a curved path using different coordinate systems.
- To present an analysis of dependent motion of two particles.
- To examine the principles of relative motion of two particles using translating axes.

Video Solutions are available for selected questions in this chapter.

12.1 Introduction

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces. Engineering mechanics is divided into two areas of study, namely, statics and dynamics. ***Statics*** is concerned with the equilibrium of a body that is either at rest or moves with constant velocity. Here we will consider ***dynamics***, which deals with the accelerated motion of a body. The subject of dynamics will be presented in two parts: *kinematics*, which treats only the geometric aspects of the motion, and *kinetics*, which is the analysis of the forces causing the motion. To develop these principles, the dynamics of a particle will be discussed first, followed by topics in rigid-body dynamics in two and then three dimensions.

Historically, the principles of dynamics developed when it was possible to make an accurate measurement of time. Galileo Galilei (1564–1642) was one of the first major contributors to this field. His work consisted of experiments using pendulums and falling bodies. The most significant contributions in dynamics, however, were made by Isaac Newton (1642–1727), who is noted for his formulation of the three fundamental laws of motion and the law of universal gravitational attraction. Shortly after these laws were postulated, important techniques for their application were developed by Euler, D'Alembert, Lagrange, and others.

There are many problems in engineering whose solutions require application of the principles of dynamics. Typically the structural design of any vehicle, such as an automobile or airplane, requires consideration of the motion to which it is subjected. This is also true for many mechanical devices, such as motors, pumps, movable tools, industrial manipulators, and machinery. Furthermore, predictions of the motions of artificial satellites, projectiles, and spacecraft are based on the theory of dynamics. With further advances in technology, there will be an even greater need for knowing how to apply the principles of this subject.

Problem Solving. Dynamics is considered to be more involved than statics since both the forces applied to a body and its motion must be taken into account. Also, many applications require using calculus, rather than just algebra and trigonometry. In any case, the most effective way of learning the principles of dynamics is ***to solve problems***. To be successful at this, it is necessary to present the work in a logical and orderly manner as suggested by the following sequence of steps:

1. Read the problem carefully and try to correlate the actual physical situation with the theory you have studied.
2. Draw any necessary diagrams and tabulate the problem data.
3. Establish a coordinate system and apply the relevant principles, generally in mathematical form.
4. Solve the necessary equations algebraically as far as practical; then, use a consistent set of units and complete the solution numerically. Report the answer with no more significant figures than the accuracy of the given data.
5. Study the answer using technical judgment and common sense to determine whether or not it seems reasonable.
6. Once the solution has been completed, review the problem. Try to think of other ways of obtaining the same solution.

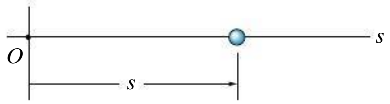
In applying this general procedure, do the work as neatly as possible. Being neat generally stimulates clear and orderly thinking, and vice versa.

12.2 Rectilinear Kinematics: Continuous Motion

We will begin our study of dynamics by discussing the kinematics of a particle that moves along a rectilinear or straight-line path. Recall that a *particle* has a mass but negligible size and shape. Therefore we must limit application to those objects that have dimensions that are of no consequence in the analysis of the motion. In most problems, we will be interested in bodies of finite size, such as rockets, projectiles, or vehicles. Each of these objects can be considered as a particle, as long as the motion is characterized by the motion of its mass center and any rotation of the body is neglected.

Rectilinear Kinematics. The kinematics of a particle is characterized by specifying, at any given instant, the particle's position, velocity, and acceleration.

Position. The straight-line path of a particle will be defined using a single coordinate axis s, Fig. 12–1*a*. The origin O on the path is a fixed point, and from this point the ***position coordinate*** s is used to specify the location of the particle at any given instant. The magnitude of s is the distance from O to the particle, usually measured in meters (m), and the sense of direction is defined by the algebraic sign on s. Although the choice is arbitrary, in this case s is positive since the coordinate axis is positive to the right of the origin. Likewise, it is negative if the particle is located to the left of O. Realize that *position is a vector quantity* since it has both magnitude and direction. Here, however, it is being represented by the algebraic scalar s, rather than in boldface $\mathbf{s}$, since the direction always remains along the coordinate axis.

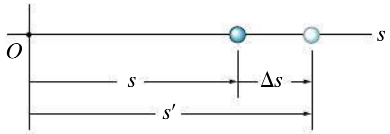


Position

(a)

Displacement. The ***displacement*** of the particle is defined as the *change in its position*. For example, if the particle moves from one point to another, Fig. 12–1*b*, the displacement is

$$\Delta s = s' - s$$



Displacement

(b)

Fig. 12–1

In this case Δs is *positive* since the particle's final position is to the *right* of its initial position, i.e., $s' > s$. Likewise, if the final position were to the *left* of its initial position, Δs would be *negative*.

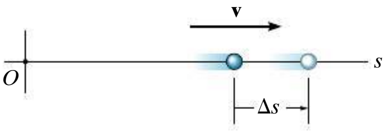
The displacement of a particle is also a *vector quantity*, and it should be distinguished from the distance the particle travels. Specifically, the *distance traveled* is a *positive scalar* that represents the total length of path over which the particle travels.

Velocity. If the particle moves through a displacement Δs during the time interval Δt, the ***average velocity*** of the particle during this time interval is

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t}$$

If we take smaller and smaller values of Δt, the magnitude of Δs becomes smaller and smaller. Consequently, the ***instantaneous velocity*** is a vector defined as $v = \lim_{\Delta t \to 0} (\Delta s / \Delta t)$, or

$$(\overset{+}{\rightarrow}) \qquad \boxed{v = \frac{ds}{dt}} \qquad (12\text{–}1)$$



Velocity

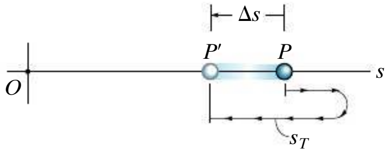
(c)

Since Δt or dt is always positive, the sign used to define the *sense* of the velocity is the same as that of Δs or ds. For example, if the particle is moving to the *right*, Fig. 12–1*c*, the velocity is *positive;* whereas if it is moving to the *left*, the velocity is *negative*. (This is emphasized here by the arrow written at the left of Eq. 12–1.) The *magnitude* of the velocity is known as the ***speed***, and it is generally expressed in units of m/s.

Occasionally, the term "average speed" is used. The ***average speed*** is always a positive scalar and is defined as the total distance traveled by a particle, s_T, divided by the elapsed time Δt; i.e.,

$$(v_{\text{sp}})_{\text{avg}} = \frac{s_T}{\Delta t}$$

For example, the particle in Fig. 12–1*d* travels along the path of length s_T in time Δt, so its average speed is $(v_{\text{sp}})_{\text{avg}} = s_T/\Delta t$, but its average velocity is $v_{\text{avg}} = -\Delta s/\Delta t$.



Average velocity and
Average speed

(d)

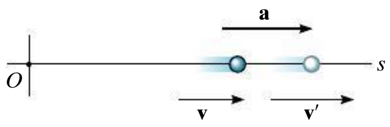
Fig. 12–1 (cont.)

Acceleration. Provided the velocity of the particle is known at two points, the ***average acceleration*** of the particle during the time interval Δt is defined as

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

Here Δv represents the difference in the velocity during the time interval Δt, i.e., $\Delta v = v' - v$, Fig. 12–1e.

The ***instantaneous acceleration*** at time t is a *vector* that is found by taking smaller and smaller values of Δt and corresponding smaller and smaller values of Δv, so that $a = \lim_{\Delta t \to 0} (\Delta v/\Delta t)$, or



Acceleration

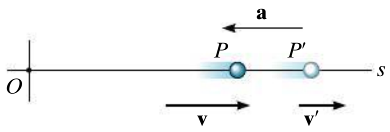
(e)

$$(\overset{+}{\rightarrow}) \qquad \boxed{a = \frac{dv}{dt}} \qquad (12\text{–}2)$$

Substituting Eq. 12–1 into this result, we can also write

$$(\overset{+}{\rightarrow}) \qquad a = \frac{d^2s}{dt^2}$$

Both the average and instantaneous acceleration can be either positive or negative. In particular, when the particle is *slowing down*, or its speed is decreasing, the particle is said to be ***decelerating***. In this case, v' in Fig. 12–1f is *less* than v, and so $\Delta v = v' - v$ will be negative. Consequently, a will also be negative, and therefore it will act to the *left*, in the *opposite sense* to v. Also, notice that if the particle is originally at rest, then it can have an acceleration if a moment later it has a velocity v'; and, if the *velocity* is *constant*, then the *acceleration is zero* since $\Delta v = v - v = 0$. Units commonly used to express the magnitude of acceleration are m/s^2.



Deceleration

(f)

Fig. 12–1 (cont.)

Finally, an important differential relation involving the displacement, velocity, and acceleration along the path may be obtained by eliminating the time differential dt between Eqs. 12–1 and 12–2. We have

$$dt = \frac{ds}{v} = \frac{dv}{a}$$

or

$$(\overset{+}{\rightarrow}) \qquad \boxed{a\,ds = v\,dv} \qquad (12\text{–}3)$$

Although we have now produced three important kinematic equations, realize that the above equation is not independent of Eqs. 12–1 and 12–2.

When the ball is released, it has zero velocity but an acceleration of 9.81 m/s^2.

Constant Acceleration, $a = a_c$. When the acceleration is constant, each of the three kinematic equations $a_c = dv/dt$, $v = ds/dt$, and $a_c\,ds = v\,dv$ can be integrated to obtain formulas that relate a_c, v, s, and t.

Velocity as a Function of Time. Integrate $a_c = dv/dt$, assuming that initially $v = v_0$ when $t = 0$.

$$\int_{v_0}^{v} dv = \int_0^t a_c\,dt$$

$$(\overset{+}{\rightarrow}) \qquad \boxed{\begin{array}{c} v = v_0 + a_c t \\ \text{Constant Acceleration} \end{array}} \qquad (12\text{–}4)$$

Position as a Function of Time. Integrate $v = ds/dt = v_0 + a_c t$, assuming that initially $s = s_0$ when $t = 0$.

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + a_c t)\,dt$$

$$(\overset{+}{\rightarrow}) \qquad \boxed{\begin{array}{c} s = s_0 + v_0 t + \frac{1}{2} a_c t^2 \\ \text{Constant Acceleration} \end{array}} \qquad (12\text{–}5)$$

Velocity as a Function of Position. Either solve for t in Eq. 12–4 and substitute into Eq. 12–5, or integrate $v\,dv = a_c\,ds$, assuming that initially $v = v_0$ at $s = s_0$.

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a_c\,ds$$

$$(\overset{+}{\rightarrow}) \qquad \boxed{\begin{array}{c} v^2 = v_0^2 + 2a_c(s - s_0) \\ \text{Constant Acceleration} \end{array}} \qquad (12\text{–}6)$$

The algebraic signs of s_0, v_0, and a_c, used in the above three equations, are determined from the positive direction of the s axis as indicated by the arrow written at the left of each equation. Remember that these equations are useful *only when the acceleration is constant and when* $t = 0$, $s = s_0$, $v = v_0$. A typical example of constant accelerated motion occurs when a body falls freely toward the earth. If air resistance is neglected and the distance of fall is short, then the *downward* acceleration of the body when it is close to the earth is constant and approximately 9.81 m/s^2. The proof of this is given in Example 13.2.

Important Points

- Dynamics is concerned with bodies that have accelerated motion.
- Kinematics is a study of the geometry of the motion.
- Kinetics is a study of the forces that cause the motion.
- Rectilinear kinematics refers to straight-line motion.
- Speed refers to the magnitude of velocity.
- Average speed is the total distance traveled divided by the total time. This is different from the average velocity, which is the displacement divided by the time.
- A particle that is slowing down is decelerating.
- A particle can have an acceleration and yet have zero velocity.
- The relationship $a\,ds = v\,dv$ is derived from $a = dv/dt$ and $v = ds/dt$, by eliminating dt.

During the time this rocket undergoes rectilinear motion, its altitude as a function of time can be measured and expressed as $s = s(t)$. Its velocity can then be found using $v = ds/dt$, and its acceleration can be determined from $a = dv/dt$. (© NASA)

Procedure for Analysis

Coordinate System.

- Establish a position coordinate s along the path and specify its *fixed origin* and positive direction.
- Since motion is along a straight line, the vector quantities position, velocity, and acceleration can be represented as algebraic scalars. For analytical work the sense of s, v, and a is then defined by their *algebraic signs*.
- The positive sense for each of these scalars can be indicated by an arrow shown alongside each kinematic equation as it is applied.

Kinematic Equations.

- If a relation is known between any *two* of the four variables a, v, s, and t, then a third variable can be obtained by using one of the kinematic equations, $a = dv/dt$, $v = ds/dt$ or $a\,ds = v\,dv$, since each equation relates all three variables.*
- Whenever integration is performed, it is important that the position and velocity be known at a given instant in order to evaluate either the constant of integration if an indefinite integral is used, or the limits of integration if a definite integral is used.
- Remember that Eqs. 12–4 through 12–6 have only limited use. These equations apply *only* when the *acceleration is constant* and the initial conditions are $s = s_0$ and $v = v_0$ when $t = 0$.

*Some standard differentiation and integration formulas are given in Appendix A.